

A THEORY FOR MONODISPERSE SPRAY VAPORIZATION IN ADIABATIC AND ISOTHERMAL SYSTEMS

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Abstract—The motion and the associated phenomena of a monodisperse spray undergoing evaporation in an isothermal or adiabatic gas stream are investigated theoretically. Particular emphasis is placed on initially cold flows that may lack sufficient energy to achieve complete spray vaporization. Expressions are derived for the spacial variations of droplet velocity and size, the gas velocity and temperature, and a criterion on the initial conditions for complete spray vaporization. The relative importance of various factors affecting rapid vaporization over short distances are discussed.

NOMENCLATURE

A ,	ratio of molecular weights of inert gas to fuel;
B ,	transfer number;
C ,	heat capacity;
C_D ,	drag coefficient;
D ,	binary diffusion coefficient;
L ,	specific heat of vaporization;
M ,	non-dimensionalized mass evaporation rate;
r ,	radial distance from center of droplet;
r_c ,	radius of droplet;
\hat{r} ,	r/r_c ;
R ,	gas constant of fuel vapor;
Re ,	Reynolds number;
Sc ,	Schmidt number;
t ,	time;
T ,	temperature;
T_b ,	temperature at boiling point of fuel;
u ,	velocity;
x ,	axial distance in chamber;
Y ,	mass fraction.

Greek symbols

ϵ ,	initial mas. flux fraction of fuel;
λ ,	\hat{r}_c^2 ;
ρ ,	density;
μ ,	viscosity coefficient.

Subscripts

c ,	condensed phase;
F ,	fuel vapor;
g ,	gas phase;
0 ,	initial state;
s ,	droplet surface.

INTRODUCTION

STUDIES on spray behavior find applications in a variety of practical situations. Some examples are fire suppression, insecticide spraying, spray drying, and in many forms of chemical powerplants which introduce one or more of the reactants into the reactor in the form of spray.

The characteristics of a spray can be statistically described by a spray equation [1], which is analogous to the Boltzman equation in the kinetic theory of gases with the exception that the spray distribution function may also depend on the sizes of the droplets characterized by their radii r_c . The properties of the spray-gas ensemble can then be obtained by solving this spray equation together with the various conservation relations.

Only some simplified solutions have been obtained for this system of equations. Probert [2] analyzed the steady-state combustion of a dilute spray in a variable-area, quasi-one-dimensional liquid rocket chamber. By assuming the effects of droplets on the gas are negligible, the combustion efficiency of the chamber was determined by directly relating the amount of heat release to the mass reacted.

Spalding [3, 4] obtained analytic solutions for the combustion of monodisperse spray in a constant area liquid propellant rocket chamber. The momentum exchange process between the gas and the droplets was described by a modified Stokes law accounting, approximately, the reduction in drag due to the outward mass transfer at the droplet surface. The phase change process was described by the d^2 -law for droplet combustion, modified by the Frossling [5] correction factor to account for the increase in the burning rate due to the relative motion between the droplets and the gas. Both finite and infinite gas-phase reaction rates were considered.

The present paper extends Spalding's model to spray vaporization in an adiabatic environment. Particular attention is placed on such practical situations as in the automotive induction system where the inlet air is quite cold such that the continuous enrichment (with fuel vapor) and the chilling of the gas from the vaporization process may result in a significant reduction in the driving force for vaporization. In severe cases vaporization may terminate altogether. Incomplete fuel vaporization prior to ignition results in droplet diffusional burning in the combustion cylinders, which

could be a significant source of the total nitric oxide emissions from the engine.

The governing equations are presented in the next Section. In Sections 3 and 4, the cases of adiabatic evaporation and isothermal evaporation or combustion are discussed. Results are presented in Section 5.

2. GOVERNING EQUATIONS

This study is concerned with a steady, one-dimensional, constant area, constant density, two-phase flow involving evaporating droplets moving in a gaseous stream. For consistency, we shall term the vaporizing species as the fuel, all the other gas phase species as inert gas, and all the gas-phase species simply as gas.

At the entrance plane located at $x = 0$, droplets with uniform radius r_{c0} and velocity u_{c0} are injected evenly over the inlet cross-section into a gaseous stream with velocity u_{g0} , where the subscripts c , g and 0 designate the condensed phase, the gas phase and the initial state respectively. The initial mass flux fractions of the condensed and vaporized fuel are ϵ_c and ϵ_g respectively. The gas has a constant density ρ_g and an inlet temperature T_{g0} . Mixing between either the fuel vapor or the combustion products with the ambient gas flow is assumed to be infinitely fast such that macroscopic property uniformity exists at each cross-section of the chamber. The behavior of this two-phase system is governed by the following equations.

(a) Droplet evaporation rate

When the gas-phase diffusion of mass and heat are rate-limiting for the evaporation process, it is shown in the Appendix that the rate of decrease of the square of the droplet radius r_c is given by

$$dr_c^2/dt = -(2\rho_g D/\rho_c) \ln(1+B) \tag{1}$$

where D is the gas-phase binary diffusion coefficient, ρ_c the droplet density, and B an evaporation transfer number defined as

$$B = (C_g/L)(T_g - T_c) \tag{2}$$

with C being the specific heat at constant pressure and L the specific heat of vaporization.

Figure 8 in the Appendix illustrates that for vaporization in a relatively hot ambient gas stream, T_c can be approximated by the droplet boiling point T_b . Furthermore, since T_g also is not expected to be changed much by the vaporization process, B , as given by equation (2), is approximately a constant.

During vaporization in a relatively cold ambient gas stream, B ceases to be a constant. Under this situation the evaporation process can significantly change T_g , whereas T_c is also a sensitive function of T_g and Y_{Fg} (Fig. 8), where Y_{Fg} is the fuel vapor fraction in the gas stream.

When a relative velocity exists between the droplet and the ambient gas, Frossling [5] has shown that the droplet evaporation rate, equation (1), is increased by a factor

$$F(Re, Sc) = 1 + 0.276|Re|^{1/2}Sc^{1/3} \tag{3}$$

where $Re = 2\rho_g(u_g - u_c)r_c/\mu$ and $Sc = \mu/(\rho_g D)$ are the Reynolds and Schmidt numbers respectively, and μ is the gas-phase viscosity coefficient. The correction factor equation (3) has also been found to be applicable to burning droplets [6].

(b) The drag law

The rate of change of the droplet velocity u_c , caused by the drag force acted on it by the gas motion, is given by

$$du_c/dt = (3/8)(\rho_g/\rho_c)[(u_g - u_c)^2/r_c]C_D \tag{4}$$

where the drag coefficient C_D is given by [4],

$$C_D = [Re^{-1}K(Sc)F(Re, Sc)]G(B). \tag{5}$$

The term within the square bracket in equation (5) provides a close approximation to the experimentally determined drag coefficient curve for a solid sphere. The function $K(Sc)$ depends only on the constant parameter Sc and hence is itself a constant for a given system. It is to be determined in such a way that for a given Sc , the prescribed C_D agrees most closely with the experimentally determined values. For $Sc = 0.72$, it was found that with $K = 22$ such close agreements (Fig. 1) exist

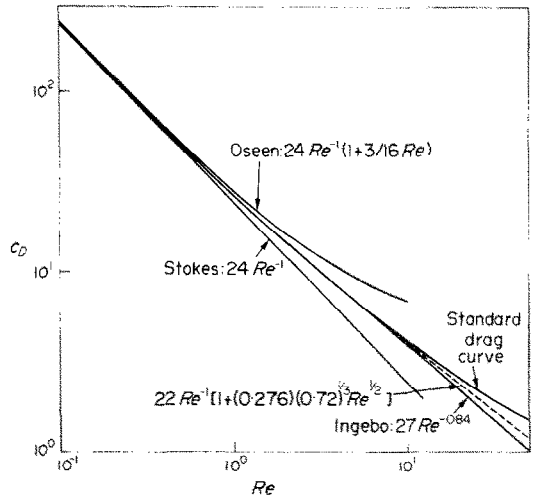


FIG. 1. Various experimental and theoretical curves for the drag coefficient of solid spheres.

with both the standard drag curve and the results of Ingebo [7]. It may also be noted that the insertion of the function $F(Re, Sc)$ into the definition of C_D not only improves the accuracy in describing the drag force on the droplets, it also simplifies the subsequent mathematical developments. Spalding [4] originally recognized the mathematical simplification offered by such a choice and incorporated it in his analysis, with $K = 24$, based on the argument of Reynolds analogy.

The second term in equation (5) accounts for the reduction in drag due to the outward mass transfer at the droplet surface during evaporation. Various forms of $G(B)$ have been suggested and experimentally tested [8], although a definitive choice is still unavailable [9]. In the following formulation when B , hence $G(B)$, can be considered a constant, the functional form of $G(B)$

is left unspecified. When B varies, in the absence of clear superiority, we use the form suggested by Spalding [4],

$$G(B) = B^{-1} \ln(1 + B) \tag{6}$$

which is the ratio of mass transfer to heat transfer, hence mass transfer to momentum transfer through Reynolds analogy, for a stationary droplet. For a slowly vaporizing droplet $B \ll 1$, hence $G(B) \simeq 1$ and the drag reduction is minimum. Further discussions on equation (5) can be found in [10].

(c) *Mass conservation*

The overall mass conservation can be expressed as

$$u_g/u_{g0} = 1 + [\varepsilon_c/(1 - \varepsilon_c)] [1 - r_c^3/r_{c0}^3]. \tag{7}$$

At this stage the functional forms of T_g and T_c , which appear in the definition for B , are still unspecified. As will be discussed in Sections 3 and 4, their relations with other flow variables depend on the energy exchange process between the condensed and gaseous phases.

(d) *General solutions*

Defining the non-dimensionalized variables as

$$\begin{aligned} \tilde{r}_c &= r_c/r_{c0}, \quad \tilde{u}_g = u_g(1 - \varepsilon_c)/u_{g0}, \quad \tilde{u}_c = u_c(1 - \varepsilon_c)/u_{g0}, \\ \tilde{t} &= 1 - t [2\rho_g D \ln(1 + B_0)] / [\rho_c r_{c0}^2], \\ \tilde{x} &= x [2(1 - \varepsilon_c)\rho_g D \ln(1 + B_0)] / [\rho_c u_{g0} r_{c0}^2], \\ S &= [G(B_0)/\ln(1 + B_0)] Sc K(Sc)/16, \end{aligned}$$

and by using the transformed variables $\omega = 1 - \tilde{r}_c^3$ and $U = \tilde{u}_g - \tilde{u}_c$, where ω and U respectively represent the mass fraction of the droplet that has vaporized at \tilde{t} , and the instantaneous relative velocity between the droplet and the gas, the governing equations become

$$d\omega/d\tilde{t} = -(3/2)(1 - \omega)^{1/3} \times [\ln(1 + B)/\ln(1 + B_0)] F(Re, Sc) \tag{8}$$

$$dU/d\tilde{t} = d\tilde{u}_g/d\tilde{t} + (3/2) \times [SU(1 - \omega)^{-2/3}/G(B_0)] F(Re, Sc) G(B) \tag{9}$$

$$\tilde{u}_g = 1 - \varepsilon_c(1 - \omega) \tag{10}$$

where

$$\begin{aligned} F(Re, Sc) &= 1 + Z |U|^{1/2} (1 - \omega)^{1/6} \\ Z &= 0.276 Re_0^{1/2} Sc^{1/3} \end{aligned}$$

and

$$Re_0 = (2\rho_g u_{g0} r_{c0}) / [\mu(1 - \varepsilon_c)].$$

The additional advantage in introducing the function $F(Re, Sc)$ into the definition of C_D now becomes apparent. Dividing equation (9) by equation (8), and using equation (10), the non-linearity in U appearing in $F(Re, Sc)$ as well as the dependence of U on Z , can be removed by using U and ω as the dependent and independent variables respectively. Furthermore, if B is only a function of ω , then $U(\omega)$ is described by a first order, linear, ordinary differential equation whose solution is

$$U(\omega) = [p(\omega)]^{-1} \left\{ \varepsilon_c \int_0^\omega p(\omega') d\omega' + U(0) \right\} \tag{11}$$

where

$$p(\omega) = \exp \left\{ \int_0^\omega [S/(1 - \omega')] [G(B)/G(B_0)] \times [\ln(1 + B_0)/\ln(1 + B)] d\omega' \right\} \tag{12}$$

and

$$U(0) = 1 - \varepsilon_c - \tilde{u}_{c0}. \tag{13}$$

Equation (11) shows that as $\omega \rightarrow 1$, $U(\omega) \rightarrow 0$ as would be expected.

Once $U(\omega)$ is determined, the location of the droplet can be found by using equation (8) and the identity

$$d\omega/d\tilde{t} = -\tilde{u}_c(d\omega/d\tilde{x}) \tag{14}$$

such that

$$\tilde{x}(\omega) = \int_0^\omega \frac{[\tilde{u}_g - U(\omega')]}{(3/2)(1 - \omega')^{1/3} F(Re, Sc)} \times \frac{\ln(1 + B_0)}{\ln(1 + B)} d\omega'. \tag{15}$$

The distance \tilde{x}^* over which complete vaporization of the droplets occurs is of particular interest and is given by

$$\tilde{x}^* = \tilde{x}(1) \tag{16}$$

It is significant to note that the initial droplet size r_{c0} does not appear in the non-dimensionalized governing equations. Therefore results from the present analysis are applicable to all values of r_{c0} . Furthermore from the definition of \tilde{x} it is seen that the chamber length x is proportional to r_{c0}^2 , hence leading to the important conclusion that fine atomization is the key factor in achieving fast vaporization over a short distance.

Finally we note that for constant Schmidt number the parameter S varies inversely with the initial transfer number B_0 . Hence large and small values of S correspond to slow and fast rates of vaporization respectively.

In the following, the functional forms of B will be specified. Two limiting and physically important cases will be examined where the solutions are relatively straightforward.

3. ADIABATIC EVAPORATION

In the present case the evaporation process continuously cools and enriches the gas stream, both of these effects, in turn, adversely affect the evaporation rate. In extreme cases when a large amount of liquid fuel is injected into a sufficiently cold gas stream, complete vaporization of the droplets may not be possible. When the gas stream temperature and fuel vapor concentration reach the corresponding dew point values, vaporization and condensation take place at equal rates and an equilibrium state is reached for the two-phase system. Results from the Appendix indicate that the evaporation process is influenced most significantly by changes in the ambient conditions when the ambient temperature is relatively cold, say $T_g/T_b < 1$.

To complete the set of governing equations, expressions are needed for the ambient fuel vapor concentration and temperature, as well as the droplet temperature as functions of say, ω .

From definition, the fuel vapor concentration is given by

$$Y_{Fg} = (\varepsilon_c \omega + \varepsilon_g)/(1 - \varepsilon_c + \varepsilon_c \omega). \quad (17)$$

Since the flow is assumed to be incompressible, hence low speed, the kinetic energy of the flow components can be neglected compared with their respective thermal energy. Conservation of energy is hence expressed as

$$\tilde{u}_g \tilde{T}_g + \varepsilon_c(1 - \omega)(\tilde{T}_c - 1) = \tilde{u}_{g0} \tilde{T}_{g0} + \varepsilon_c(\tilde{T}_{c0} - 1) \quad (18)$$

where $\tilde{T} = CT/L$, and it has been assumed that the specific heat C for the gas and liquid are the same.

Using equations (10) and (18), \tilde{T}_g can be expressed as a function of ω . Substituting Y_{Fg} and \tilde{T}_g into Y_{Fs} given by equation (A6), noting that $\tilde{T}_c \equiv \tilde{T}_s$, it is found that Y_{Fs} is only a function of \tilde{T}_c , but not ω . Hence Y_{Fs} and \tilde{T}_c can be determined by solving this function together with an independent relation between Y_{Fs} and \tilde{T}_c that describes the phase change mechanisms at the droplet surface (e.g. the Clausius-Clapeyron equation, equation (A7), for equilibrium vaporization). Since ω does not appear in both relations, we reach the conclusion that Y_{Fs} and \tilde{T}_c are constants in the system, given by

$$\tilde{T}_c = \tilde{T}_{c0} \quad (19)$$

and

$$Y_{Fs} = [B_0 + \varepsilon_g/(1 - \varepsilon_c)]/(1 + B_0) \quad (20)$$

where $B_0 = \tilde{T}_{g0} - \tilde{T}_{c0}$ is the initial evaporation transfer number.

Physically since \tilde{T}_c is elevated and lowered, respectively, by the continuous enrichment and cooling of the gas stream, the present result indicates that these two opposing effects balance each other.

With the above constant established, the tedious iterative process (see Appendix) of determining the droplet temperature variation during its motion is thus unnecessary. The subsequent mathematical analysis is greatly simplified.

The transfer number B for vaporization is then given by the relation

$$(1 + B) = (1 + B_0)[1 - \varepsilon_c \omega / (1 - \varepsilon_c + \varepsilon_c \omega)]. \quad (21)$$

Since vaporization occurs only for $B > 0$, equation (21) implies that a necessary condition for complete spray vaporization ($\omega = 1$) is

$$B_0 > \varepsilon_c / (1 - \varepsilon_c). \quad (22)$$

If equation (22) is not satisfied, evaporation ceases when $B = 0$, leaving droplets with an equilibrium size given by

$$\tilde{r}_c = [1 - B_0(1 - \varepsilon_c)/\varepsilon_c]^{1/3}. \quad (23)$$

With B given by equation (21) and by using equation (6), equation (12) can be readily integrated, giving

$$p(\omega) = \{ (1 - [\varepsilon_c / (1 - \varepsilon_c) B_0] \omega)^{(1 - \varepsilon_c)(1 + B_0)} / (1 - \omega) \}^\beta \quad (24)$$

where

$$\beta = S / [(1 - \varepsilon_c)(1 + B_0) - 1].$$

For $B_0 \gg \varepsilon_c / (1 - \varepsilon_c)$, the numerator in $p(\omega)$ can be expanded, giving $U(\omega)$, to first order approximation, as

$$U(\omega) = \left\{ \varepsilon_c \left\{ \frac{(1 - \alpha)}{(1 - \beta)} [1 - (1 - \omega)^{(1 - \beta)}] + \frac{\alpha}{(2 - \beta)} [1 - (1 - \omega)^{(2 - \beta)}] \right\} + U(0) \right\} \times \frac{(1 - \omega)^\beta}{(1 - \alpha \omega)} \quad (25)$$

where

$$\alpha = \varepsilon_c \beta / [B_0(1 + B_0)].$$

It may be noted that $\alpha > 0$, $\beta > 0$ in this case.

With $U(\omega)$ known, equations (15) and (16) then give the function $\tilde{x}(\omega)$ and the minimum length for complete vaporization, \tilde{x}^* , respectively.

4. ISOTHERMAL EVAPORATION AND COMBUSTION

A very simple solution to equation (11) can be obtained by assuming B is a constant such that

$$B = B_0 \quad (26)$$

is the needed energy equation. In addition to its application for burning sprays in liquid rocket combustion [3] where an approximately constant combustion transfer number [1] exists, equation (26) can also be used for spray vaporizing in a chamber with external heat being added in such a way that $(T_g - T_c)$ remains constant. Finally, it is also the zeroth order approximation for B in equation (21) for spray vaporization involving a small ε_c in a hot atmosphere such that the condition $B_0 \gg \varepsilon_c / (1 - \varepsilon_c)$ is satisfied.

$U(\omega)$ is now given by

$$U(\omega) = (1 - \omega)^S \{ [\varepsilon_c / (1 - S)] \times [1 - (1 - \omega)^{(1 - S)}] + U(0) \}. \quad (27)$$

It may be noted that in the present case the constant B_0 appears in the solutions only indirectly through the definition of S , which together with ε_c , Z and \tilde{u}_{c0} , completely characterize the problem. For the adiabatic evaporation case B_0 is an additional parameter.

For the special case of $F = 1$, explicit expressions can be obtained for $\tilde{x}(\omega)$ and \tilde{x}^* as

$$\tilde{x}(\lambda) = (1 - \lambda) + \frac{0.4\varepsilon_c S}{(1 - S)} (1 - \lambda^{2.5}) - \frac{U(0) + \varepsilon_c / (1 - S)}{(1 + 1.5S)} (1 - \lambda^{1 + 1.5S}) \quad (28)$$

and

$$\tilde{x}^* = [\tilde{U}_{c0} + 1.5S(1 - 0.4\varepsilon_c)] / [1 + 1.5S] \quad (29)$$

where $\lambda = \tilde{r}_c^2$.

Equation (27) indicates that $U \rightarrow U(0)$ and 0 as $S \rightarrow 0$ and ∞ respectively, implying that over most of their lifetime droplets in systems with large and small S tend to be in, and out of, phase with the gas velocity respectively.

5. RESULTS

We first present results for the isothermal case because the solutions are simpler and phenomena common to both the isothermal and adiabatic cases can thus be investigated with reduced amount of computation.

Based on the facts that $G(B_0) \ln(1+B_0) \approx B_0^{-1}$, that the values of B for vaporizing and burning droplets are usually in the ranges between 0.1 to 1 and 5 to 10 respectively, and that $ScK(Sc)/16$ is on the order of unity, the values of S are varied from 0.1 to 20 in the present investigation, with small and large values of S corresponding to burning and vaporizing droplets respectively. The range of Z is varied from 0 to 2, corresponding approximately to values of Re_0 between 0 and 50.

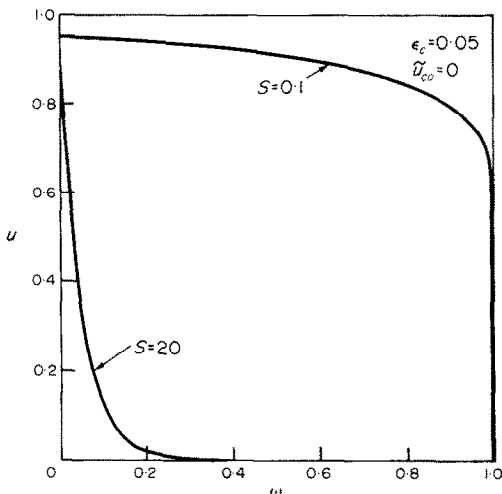


FIG. 2. Relative velocity, U , as function of the fraction of fuel vaporized, ω , for the isothermal case.

Figure 2 shows the variation of the relative velocity, U , with the amount of liquid fuel vaporized, ω , for the extreme cases of slow evaporation ($S = 20$) and fast burning ($S = 0.1$). For $S = 20$, the droplet is dragged to follow the main gas motion early in its lifetime. The relative velocity U is less than 0.01 when only 20 per cent of the mass has vaporized. For $S = 0.1$, the burning droplet lags behind the gas motion over most of its lifetime. U is still 0.8 when about 90 per cent of its mass has been consumed. During the last stage, however, the droplet adjusts rapidly to follow the mean gas motion due to the much increased drag it experiences as its size approaches zero.

Figure 3 shows the dimensionless location of the droplet as function of ω . The optimum chamber length \tilde{x}^* , for complete consumption, is significantly longer for vaporizing droplets than for burning droplets. Since the chamber length x is normalized by the evaporation rate, viz. $\ln(1+B_0)$, the difference in \tilde{x} shown in Fig. 3 exhibits only the effect of drag reduction due to the outward mass transfer at the droplet surface. Hence such a difference will be even greater when the chamber length is expressed in dimensional form since

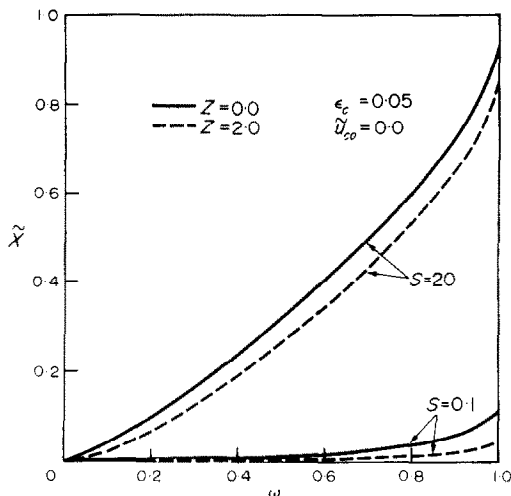


FIG. 3. Droplet location \tilde{x} as function of the fraction of fuel vaporized, ω , for the isothermal case.

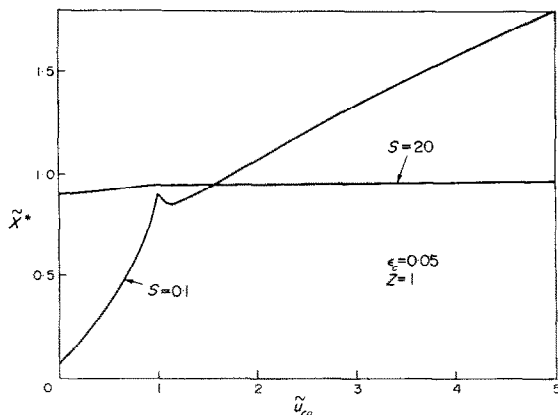


FIG. 4. Effect of droplet injection velocity \tilde{u}_{c0} on the optimum chamber length \tilde{x}^* , for the isothermal case.

$x \sim \tilde{x} / \ln(1+B_0)$. It is also shown that the droplet covers a proportionately longer distance during the last stage of its lifetime, particularly when S is small. Hence for low injection velocity flows, a significant reduction in the chamber length can be achieved by requiring that most, but not all of the droplets are vaporized.

Figure 4 shows the effect of droplet injection velocity \tilde{u}_{c0} on \tilde{x}^* . For droplets with small S values \tilde{x}^* vary approximately linearly with \tilde{u}_{c0} , indicating the absence of significant drag on the droplet motion. For large S , however, \tilde{x}^* is almost insensitive to \tilde{u}_{c0} because the droplet now adjusts rapidly to the gas motion after injection. It can thus be concluded that injection velocity is critical in the design of burners involving burning or fast evaporating droplets, and should be kept to as low a value as possible, assuming other factors remain constant. Figure 4 also shows that a local maxima in \tilde{x}^* exists when the injection velocity \tilde{u}_{c0} is near the inlet gas velocity, $\tilde{u}_{\theta 0}$, as would be expected physically. These local maxima do not exist for $Z = 0$ where \tilde{x}^* is a linear function of \tilde{u}_{c0} .

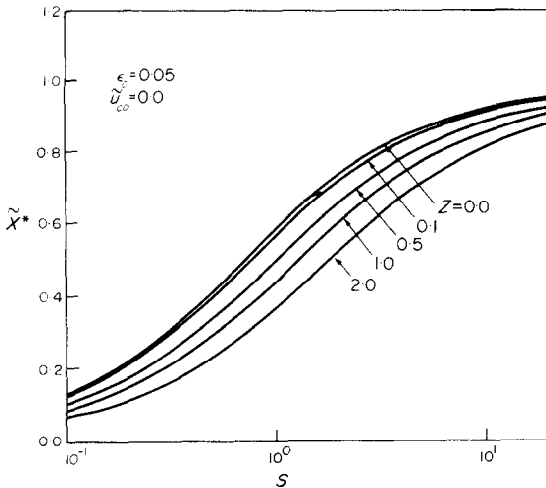


FIG. 5. Optimum chamber length \bar{x}^* as functions of S and Z , with $\epsilon_c = 0.05$, for the isothermal case.

Figure 5 shows \bar{x}^* as functions of S and Z , with zero injection velocity and for $\epsilon_c = 0.05$. It can be seen that increasing Z decreases \bar{x}^* since the droplets are now evaporating or burning faster in the presence of increased forced convection. The decrease in \bar{x}^* , however, is more prominent for small values of S . From $Z = 0$ to 2, the reductions are 8 and 50 per cent for $S = 20$ and 0.1 respectively. This is because the beneficial effect of augmenting the evaporation rate due to an increase in the initial Reynolds number Re_0 quickly disappears as $U \rightarrow 0$ for the large S case but persists throughout most of the droplet lifetime when S is small. Finally, results from Figs. 4 and 5 lead to the conclusion that for burners involving combusting or fast vaporizing droplets, an important design criterion is that the gas inlet velocity should be much greater than the droplet injection velocity.

Results of the adiabatic case are presented in the following. To completely characterize the problem, an additional parameter, B_0 , apart from S , ϵ_c , \bar{u}_{g0} and Z is needed. Noting that for the present case $S = ScK(Sc)/(16B_0)$, and using $Sc = 0.72$ and $K = 22$, we obtain $B_0 = 0.99/S$. Results presented below are for this special case, which is usually valid in many practical solutions.

Figure 6 shows \bar{x}^* as functions of S and Z for $\bar{u}_{c0} = 0$ and $\epsilon_c = 0.05$. For $S > 1$, due to the continuous decrease in the gas temperature, \bar{x}^* is significantly longer than the corresponding situation in the isothermal case. As required by equation (22), a maximum S , hence a minimum B_0 , exists beyond which complete vaporization is not possible. As S approaches this value \bar{x}^* grows exponentially large. Since B_0 is extremely small under these situations, a small increment in the inlet gas temperature T_{g0} can produce a substantial increase in B_0 and hence a significant reduction in \bar{x}^* .

For small values of S such that $B_0 \gg \epsilon_c/(1 - \epsilon_c)$, the adiabatic results are similar to those obtained for the isothermal case, as expected.

Figure 7 shows the location of the droplet as function of ω for $S = 15$ and $\epsilon_c = 0.05$. It is seen that the droplet

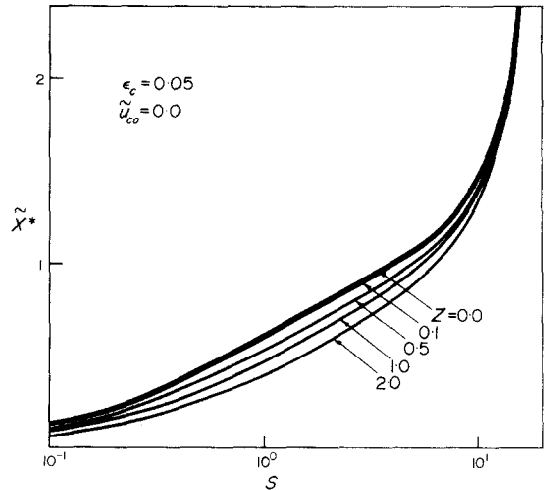


FIG. 6. Optimum chamber length \bar{x}^* as functions of S and Z , with $\epsilon_c = 0.05$, for the adiabatic case.

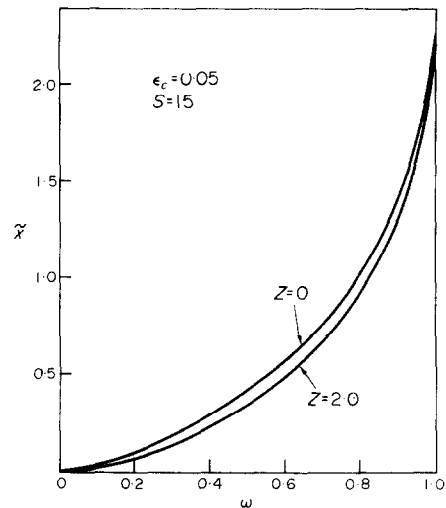


FIG. 7. Droplet location \bar{x} as function of the fraction of fuel vaporized, ω , for the adiabatic case.

covers a substantial fraction of \bar{x}^* during its last stage of evaporation when the gas stream becomes very cold. Hence a significant reduction in the chamber length can be achieved if one aims for slightly less than complete spray vaporization in the chamber. In Fig. 7, for $Z = 0$, 25 and 40 per cent reductions in the chamber length can be achieved by aiming to vaporize only 95 and 90 per cent of the spray respectively.

6. CONCLUSIONS

The motion of an ensemble of monosize droplets undergoing evaporation in an adiabatic or isothermal gas stream has been investigated. It is shown that the temperature of the droplets remain constant during their motion, that fine atomization is a key factor in achieving fast vaporization of the spray, that for rapidly and slowly vaporizing droplets the values of \bar{x}^* depend sensitively on the injection velocity and the inlet gas temperature respectively, and that a significant reduction in the chamber length can be achieved by aiming to vaporize most, but not all, of the spray.

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APPENDIX

Quasi-Steady Evaporation of Droplets

We consider herein the spherically symmetric, quasi-steady, isobaric, gas-phase mass and heat diffusion controlled evaporation of a droplet in a quiescent, unbounded atmosphere with known properties T_g and Y_{Fg} . By further assuming that the various transport coefficients and specific heats are constant, and that the Lewis number is unity, the non-dimensionalized diffusion equations for the fuel vapor concentration Y_F and temperature T are [1]

$$dY_F/d\hat{r} = -M(1 - Y_F)/\hat{r}^2 \tag{A1}$$

$$d\hat{T}/d\hat{r} = M(1 + \hat{T} - \hat{T}_s)/\hat{r}^2 \tag{A2}$$

where $M = -[\rho_c/(2\rho_g D)] dr_s^2/dt$ and $\hat{r} = r/r_s$ are the non-dimensionalized mass evaporation rate and radial distance r from the center of the droplet respectively, and the subscript s designates conditions at the droplet surface such that r_s and T_s here respectively correspond to r_c and T_c in the main text.

Integrating equations (A1) and (A2) between \hat{r} and the ambient gas conditions at infinity, the spacial distributions of the fuel vapor and temperature are found to be

$$Y_F = 1 - (1 - Y_{Fg}) \exp(-M/\hat{r}) \tag{A3}$$

$$\hat{T} = -(1 - \hat{T}_s) + (1 + \hat{T}_g - \hat{T}_s) \exp(-M/\hat{r}) \tag{A4}$$

Evaluating equations (A3) and (A4) at the droplet surface and solving, the expressions for M and the fuel vapor concentration at the droplet surface, Y_{Fs} , are given by

$$M = \ln(1 + \hat{T}_g - \hat{T}_s) \tag{A5}$$

$$Y_{Fs} = 1 - (1 - Y_{Fg})/(1 + \hat{T}_g - \hat{T}_s) \tag{A6}$$

in terms of one unknown, \hat{T}_s . Equation (A5) is identical to equation (1) with B given by equation (2).

An independent relation between \hat{T}_s and Y_{Fs} can be obtained through a description of the phase change process at the droplet surface. For equilibrium vaporization such a relation is given by the Clausius–Clapeyron equation,

$$Y_{Fs} = \{(1 - A) + A \exp[-(\tilde{C}_g/\hat{T}_s) + (\tilde{C}_g/\hat{T}_s)]\}^{-1} \tag{A7}$$

where $\tilde{C}_g = C_g/R$, A is the ratio of the average molecular weight of the inert gas to the molecular weight of the fuel, and R is the gas constant of the fuel. Equating equations (A6) and (A7), \hat{T}_s can be solved iteratively.

Figure 8 shows a typical set of solutions for water droplets vaporizing in air. To the left and right of the dew point line, corresponding to $M = 0$, condensation and vaporization occur respectively. It is shown that in a relatively cold ambient atmosphere, say $T_g/T_b < 1$, T_s , hence the rate of vaporization, is a sensitive function of the ambient temperature and fuel vapor concentration; the latter in particular. The sensitivity decreases rapidly as T_g increases. Hence for vaporization in a hot environment, say $T_g/T_b > 2$, $B = \hat{T}_g - \hat{T}_s$ can be essentially treated as a constant, independent of variations in ambient conditions. Finally we note that when $T_g/T_b \gg 1$, $T_s/T_b \approx 1$.

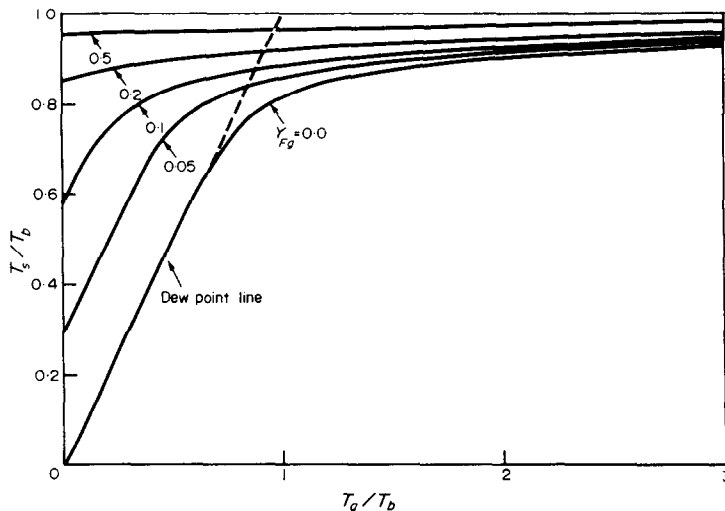


FIG. 8. Droplet temperature as functions of ambient temperature and vapor concentration for water droplets vaporizing in air.

THEORIE DE LA VAPORISATION D'UN BROUILLARD PULVERISE MONODISPERSE
DANS LES SYSTEMES ADIABATIQUES ET ISOTHERMES

Résumé—On étudie par voie théorique le mouvement et le phénomène associé d'évaporation à laquelle est soumis un brouillard pulvérisé monodispersé dans un courant gazeux isotherme ou adiabatique. Une importance particulière est donnée aux écoulements initialement froids auxquels peut manquer l'énergie suffisante pour une vaporisation complète du brouillard. On a obtenu des expressions pour les variations spatiales des vitesses et dimensions des gouttelettes, la vitesse et la température du gaz, et un critère portant sur les conditions initiales assurant une vaporisation complète du brouillard. L'importance relative des divers facteurs qui affectant la vaporisation rapide sur les courtes distances fait l'objet d'une discussion.

EINE THEORIE ZUR VERDAMPFUNG VON MONODISPERSEM SPRAY
IN ADIABATEN UND ISOTHERMEN SYSTEMEN

Zusammenfassung—Die Bewegung und das damit zusammenhängende Phänomen der Verdampfung eines monodispersen Sprays in einem isothermen oder adiabaten Gasstrom werden theoretisch untersucht. Es wird besonders betont, daß die kalte Anfangsströmung nicht genügend Energie besitzt zur vollständigen Sprayverdampfung. Es werden Beziehungen abgeleitet für die räumliche Verteilung der Tröpfchengeschwindigkeit und -größe, der Geschwindigkeit und Temperatur des Gases und ein Kriterium für die Anfangsbedingungen für vollständige Sprayverdampfung. Es wird die relative Bedeutung von verschiedenen Faktoren diskutiert, die eine schnelle Verdampfung über kurze Distanzen bewirken.

ТЕОРИЯ МОНОДИСПЕРСНОГО РАСПЫЛИТЕЛЬНОГО ИСПАРЕНИЯ В
АДИАБАТИЧЕСКИХ И ИЗОТЕРМИЧЕСКИХ СИСТЕМАХ

Аннотация—Для струи монодисперсных капель, испаряющихся в изотермическом или адиабатическом потоке газа, проведено теоретическое исследование движения и других связанных с этим процессом явлений.

Особо обращено внимание на случай, когда поток вначале является холодным и не обладает количеством энергии, необходимым для полного испарения капель. Выведены выражения для пространственных изменений скорости и размера капель, скорости и температуры газа и критерий начальных условий полного испарения струи. Рассмотрена относительная значимость различных факторов, влияющих на быстрое испарение на малых расстояниях.